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Two Preisach type vector hysteresis models

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Abstract

Two isotropic vector hysteresis models are constructed based on the classical Preisach model, to be applied in magnetostatic computations with an integral equation formulation. One model endows complete Preisach operators with orientation governed by a friction-like mechanism. The other approach—a natural generalization of the classical Preisach model—considers a set of vector operators with various coercive fields, mean interaction field (shifted spheres of various radii) and weights. The principles of the inclusion of these models in magnetostatic computations are outlined.

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Keywords: Vector hysteresis modelling; Preisach model

1. Introduction

In the general case (and in most cases of technical interest) the magnetic field strength and magnetization vectors are not parallel to each other inside a ferromagnetic body and vector hysteresis models are required to describe the behavior of the material under the given conditions of excitation.

The classical Preisach model [1] can accurately and efficiently (from the computational point of view) describe scalar hysteresis, where the magnetic field strength is always parallel to magnetization. This is the case of stacked toroidal cores of isotropic iron sheets, cast iron rings or infinite sheets excited along a fixed direction, parallel to the surface. The Preisach model also allows various vector generalizations [1–3].

Vector study is required when the field strength and magnetization vectors are not parallel. This occurs in isotropic materials due to remanence (anisotropy induced by magnetic history) while anisotropic materials are intrinsically characterized by different magnetic properties in different directions [4]. The highly non-linear, multivalued vector hysteresis operator can then be applied for material description in magnetic field computation algorithms based on contraction iteration (fixed point) techniques [5].

2. The friction model

The first proposed isotropic vector hysteresis model consists of n scalar (classical) Preisach

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operators endowed with orientation governed by a friction-like mechanism. Hysteresis itself is characterized by the effect lagging behind the cause. In the case of the classical Preisach model this is realized by the elementary operators with rectangular characteristic, which again is a friction-like feature. An assembly of scalar operators, each following the input variation with a different lag describes vector magnetizing processes.

The magnitude of the output of individual operators is computed by the classical Preisach model with the projection of field strength on their direction as input ($H\cos\theta_k$).

$$M_k = \mathrm{H}(H\cos\theta_k) = \mathrm{H}(\boldsymbol{H}\cdot\boldsymbol{M}_k), \quad k = 0, 1, \cdots, n \tag{1}$$

where H stands for the scalar hysteresis operator, H is the applied field strength and θ_k is the angle between the field strength and the magnetization vector of the operator. This angle is determined by the equilibrium between a driving torque $\boldsymbol{H}\cdot\boldsymbol{M}_k$ tending to rotate magnetization towards field strength and a friction-like resistive torque (mainly due to pinning [6]), proportional to $M_k^{3/2}(M_S - M_k)^{1/2}$ with M_S being the value of magnetization. This damping term is chosen in order to comply with the experimental fact that at saturation the magnetization vector rotates together with field strength. The orientations of the magnetization vectors change when the driving torque overcomes the resisting one, leading to:

$$|\sin\theta_k| \leqslant \xi_k \frac{H_S}{|H|}\sqrt{\frac{|M_k|}{M_S}\left(1 - \frac{|M_k|}{M_S}\right)}, \quad k = 0, n, \tag{2}$$

where H_S is the saturation field strength. The "friction coefficients" ξ_k are different for each operator and the weighted sum of the outputs of these individual operators is the output of the vector hysteresis operator—the averaged magnetization in the region of space characterized by it [7].

The magnetization vector lies in the plane defined by the new position of the applied field strength and the old position of the magnetization, obeying:

$$\boldsymbol{M}'_k \cdot (\boldsymbol{H}\cdot\boldsymbol{M}_k) = 0,$$
$$\frac{\boldsymbol{M}'_k \cdot \boldsymbol{H}}{M'_k H} = \cos\theta_k, \tag{3}$$

where $\boldsymbol{M}_k$ is the old, $\boldsymbol{M}'_k$ the new magnetization vector of the operator k and θ_k is determined according to Eq. 2. In the two-dimensional case (2D), Eq. (2) is sufficient to determine the position of the magnetization vector, since it always lies in the same plane.

Figs. 1–4 illustrate the operation of the model in the 2D case. Fig. 1 shows the scalar major loop of the considered material, Figs. 2 and 3, illustrate the result of rotational magnetization subsequent to saturation in one direction (x) and reduction of field strength to some value, while Fig. 4 depicts linear magnetization subsequent to saturation in an orthogonal direction (illustrating remanence-induced anisotropy). The thin gray lines connect points of the $\boldsymbol{M}$ and $\boldsymbol{H}$ plots occurring simultaneously.

Uniaxial anisotropy can be introduced by means of an anisotropy field oriented along the easy axis, which gives a further term in the torque balance [8].

3. The coercive spheres model

In analogy with the classical Preisach model, a 3D vector model can be constructed as a set of elementary vector operators. These operators are defined by a mean interaction field $\boldsymbol{H}_m^k$ and coercivity H_c^k. The output of such an operator is

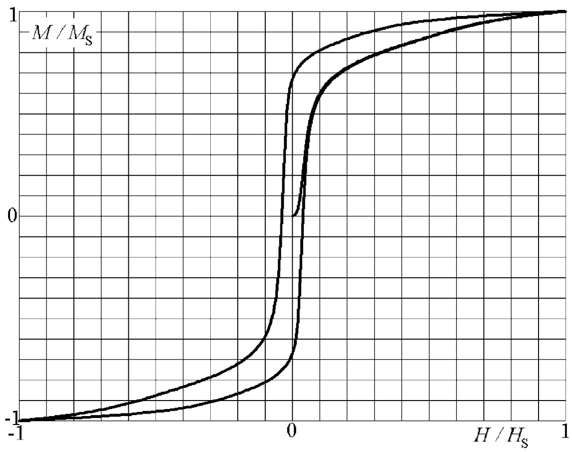

Fig. 1. Major scalar hysteresis loop.

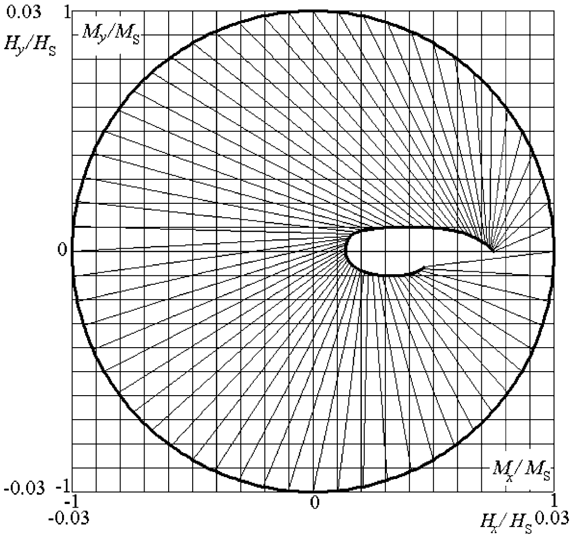

Fig. 2. Rotational magnetization subsequent to saturation in one direction without magnetization reversal.

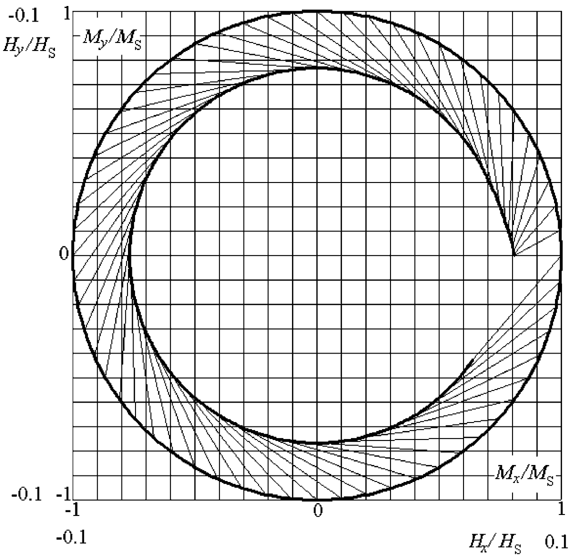

Fig. 3. Rotational magnetization subsequent to saturation in one direction.

a vector $\boldsymbol{M}^k$ with constant magnitude. It is parallel to $\boldsymbol{H} - \boldsymbol{H}_\mathrm{m}^k$ if $\boldsymbol{H}$ is outside the sphere of radius H_c^k centered at $\boldsymbol{H}_\mathrm{m}^k$ (Fig. 5) and preserves the orientation it had at the moment it entered the sphere as long as the tip of $\boldsymbol{H}$ lies inside it.

A distribution of such operators with respect to $\boldsymbol{H}_\mathrm{m}^k$ and H_c^k is considered similar to the case of the

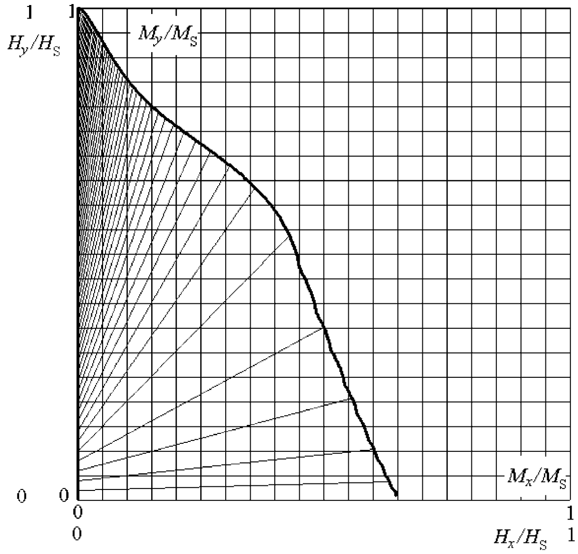

Fig. 4. Remanence-induced anisotropy.

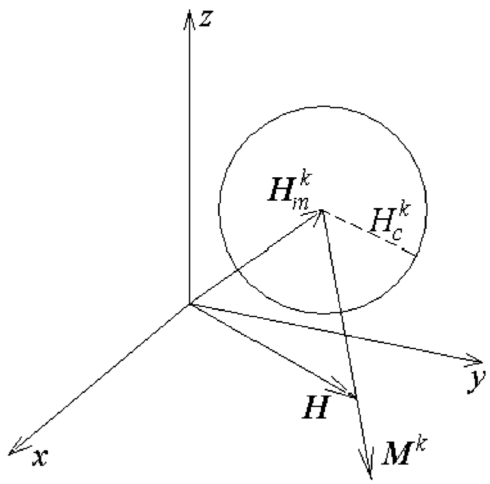

Fig. 5. Coercive sphere operator.

classical Preisach model. The output of the model is the weighted sum of the elementary operator outputs.

Figs. 6 and 7 illustrate the operation of the model in two dimensions (where the spheres are replaced by circles) for a simple configuration of two rings of 36 operators each, with $H_\mathrm{c} = 15$, $H_\mathrm{m} = 10$ and $H_\mathrm{c} = 12$, $H_\mathrm{m} = 15$, respectively. In Fig. 6 the major loop for a unidirectional

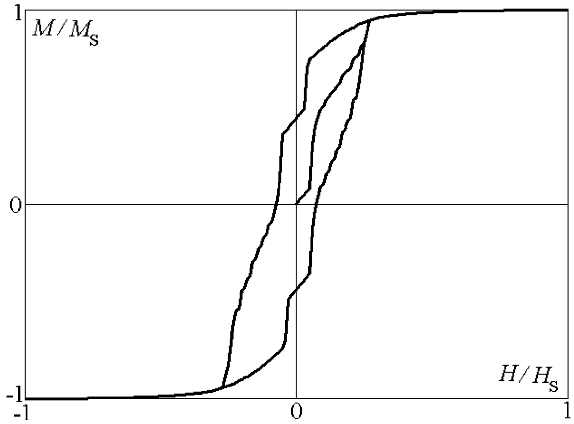

Fig. 6. Major loop for alternating excitation.

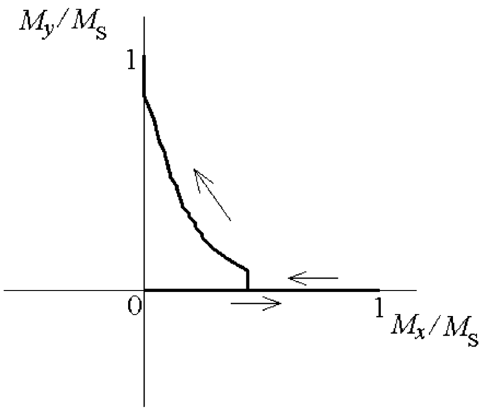

Fig. 7. Remanence-induced anisotropy.

alternating excitation is plotted, while Fig. 7 depicts the trace of the tip of $\boldsymbol{M}$ at linear magnetization subsequent to saturation in an orthogonal direction (illustrating remanence-induced anisotropy).

4. Inclusion in magnetostatic computations

The integral equation method can be applied for the computation of magnetic fields in large areas, in the presence of ferromagnetic bodies. Only the bodies are divided into uniformly magnetized elements and coefficient matrices $[\mathbf{C}_{kl}]$ constructed, which give the influence of the element magnetizations on each other:

$$\boldsymbol{H}_k = \boldsymbol{H}_0 + \sum_{l=1}^{N} [\mathbf{C}_{kl}]\boldsymbol{M}_l, \quad k = 1, N, \tag{4}$$

where $\boldsymbol{M}_l$ is the magnetization of element l, $\boldsymbol{H}_0$ is the external field in the center of element k, $\boldsymbol{H}_k$ being the field strength in the same place.

Then the magnetizations of the elements are computed iteratively, so that the $\boldsymbol{M}-\boldsymbol{H}$ relationship on each element fulfills the magnetic characteristic of the material. As for the iterative method, a fixed-point technique [5], non-linear minimization of an error function:

$$\varepsilon = \sum_{k=1}^{N} |\boldsymbol{M}_k - \tilde{\boldsymbol{M}}_k|^2 \rightarrow \min, \quad \tilde{\boldsymbol{M}}_k = \boldsymbol{H}(\boldsymbol{H}_k), \tag{5}$$

or a contraction iteration can be applied:

$$\boldsymbol{M}_k^{i+1} = \tau \boldsymbol{M}_k^i + (1-\tau)\tilde{\boldsymbol{M}}_k^i, \quad 0 < \tau \ll 1. \tag{6}$$

In Eq. (5) and (6) $\boldsymbol{M}_k$ is the predicted, $\tilde{\boldsymbol{M}}_k$ the computed value of the magnetization vector. These methods can also be combined (e.g. when the error minimization gets stuck in a local minimum, the contraction iteration can bounce the process out of such traps).

5. Conclusions

The described vector hysteresis models, intended to be included in magnetostatic computation codes, yield results in qualitative agreement with the experiment. Further advantages are the memory economy and high computational speed. A trade-off is sought between the higher accuracy of more complex models and the computational efficiency of the classical Preisach model.

Acknowledgements

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