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Physics of the Earth and Planetary Interiors 154 (2006) 299–307

PHYSICS OF THE EARTH AND PLANETARY INTERIORS

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Approach to saturation analysis of hysteresis measurements in rock magnetism and evidence for stress dominated magnetic anisotropy in young mid-ocean ridge basalt

Karl Fabian*

Universität Bremen, FB Geowissenschaften, Postfach 330440, 28334 Bremen, Germany

Received 2 December 2004; received in revised form 24 May 2005; accepted 7 November 2005

Abstract

Young mid-ocean ridge basalts contain titanomagnetite crystals of varying size and composition. Many studies of their hysteresis properties have found M_{rs}/M_s ratios considerably above the theoretical limit of 0.5 for uniaxial single domain particles. Since titanomagnetite is a cubic mineral, high M_{rs}/M_s could occur due to cubic anisotropy which allows for M_{rs}/M_s values up to 0.866. On the other hand, titanomagnetites with high Ti content possess extremely large magnetostriction constants. Already slight internal stress easily outweighs cubic anisotropy and enforces uniaxial behavior. Are high M_{rs}/M_s ratios now a proof for very low internal stress? No! On the contrary, previous studies on synthetic titanomagnetite with high M_{rs}/M_s show that after annealing this ratio decreases. A possible explanation is that insufficient saturation of the hysteresis loop, used to infer M_{rs}/M_s, leads to underestimation of M_s. Here, a systematic experimental study on a young mid-ocean ridge basalt using fields of up to 7 T demonstrates that indeed the M_{rs}/M_s ratio of the single domain fraction does not significantly deviate from the theoretical value of 0.5 for uniaxial anisotropy. It is further estimated that internal stress above 200 MPa is necessary to explain the observed hysteresis behavior – a value which is consistent with recent independent approximations. On the other hand, theoretical loops for cubic minerals do not fit the observed data. In order to assess the validity of M_s determinations from hysteresis measurements, an improved method to evaluate the approach to saturation behavior of hysteresis loops is developed. It allows to recognize insufficiently saturated loops and thereby helps to avoid misinterpretation of standard M_{rs}/M_s measurements when high fields are not accessible.

Keywords: Rock magnetism; Hysteresis; Mid ocean ridge basalt; Stress; Saturation magnetization

1. Introduction

For mid-ocean ridge basalt (MORB) extremely high ratios of saturation remanence M_{rs} to saturation magnetization M_s, sometimes above 0.7, are often reported (e.g. Day et al., 1978; Gee and Kent, 1995). This indicates that remanence originates from stable single-domain (SD) particles because remanence carriers with inhomogeneous or thermally instable magnetization structure lead to much lower M_{rs}/M_s ratios. Even more puzzling, isotropic ensembles of stable uniaxial SD particles have an upper limit of $M_{rs}/M_s = 0.5$ (Stoner and Wohlfarth, 1948) and microscopic evidence even suggests the presence of multidomain remanence carriers also in MORB with high M_{rs}/M_s (Gee and Kent, 1995). For MORB samples with M_{rs}/M_s ratios greater 0.5, this seems to provide ample evidence to conclude that uniaxial anisotropy cannot be important here. As an alter-

* Tel.: +49 421 218 3938; fax: +49 421 218 7008.
E-mail address: karl.fabian@uni-bremen.de.

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doi:10.1016/j.pepi.2005.06.016

Table 1
Theoretical values of h_c, h_{cr} in units of $2\,|K|/M_s$ and M_{rs}/M_s for isotropic non-interacting SD particle ensembles of different anisotropies, where K denotes either K_u for uniaxial or K_1 for cubic anisotropy (Joffe and Heuberger, 1974)

	Prolate	Oblate	Cubic: $K_1 < 0$	Cubic: $K_1 > 0$
h_c	0.479	0.562	0.189	0.321
h_{cr}	0.524	0.648	0.204	0.333
M_{rs}/M_s	0.5	0.638	0.866	0.831

native, it was suggested that in these samples the cubic crystallographic anisotropy of titanomagnetite may prevail (Gee and Kent, 1995). According to Table 1 this could explain for M_{rs}/M_s ratios up to 0.866 if $K_1 < 0$.

From a different point of view, however, predominant cubic anisotropy in titanomagnetite TMx with high ulvöspinel content $x \approx 60\%$ is quite unexpected. As compared to magnetite with $x = 0$, data from synthetic TM60 crystals report a lower magnetocrystalline anisotropy constant K_1 and much larger moduli of the magnetostriction constants λ_{100} and λ_{111} (Syono and Ishikawa, 1963; Klerk et al., 1977). In combination with moderate internal stress this preferably generates dominant uniaxial anisotropy. Fig. 1 extends an analysis of Appel and Soffel (1984) in comparing the characteristic magnetic energy contributions from anisotropy, strayfield and magnetostriction to the magnetoelastic energy in function of stress σ and ulvöspinel content x in the titanomagnetite TMx solid solution series. It is based on the energy estimates collected in Table 2 and indicates for each ulvöspinel content x the critical stress values where K_σ equals the characteristic energy densities $|K_1|/3$, K_d and K_{ms}. The ratio $Q = K/K_d$ of leading anisotropy energy K and demagnetizing energy determines the magnetic hardness of a material. If $Q \ll 1$, as in magnetite, the material is considered as magnetically soft which implies that closure domains or fine scale domain structures can exist.

The regions outlined in Fig. 1 approximately classify source and strength of magnetic hardness in function of external stress. For low Ti content $x < 0.37$ and $\sigma <\sim 10$ MPa the material is magnetically soft and external stress does not overcome cubic anisotropy. For higher σ the cubic domain structure breaks down, and

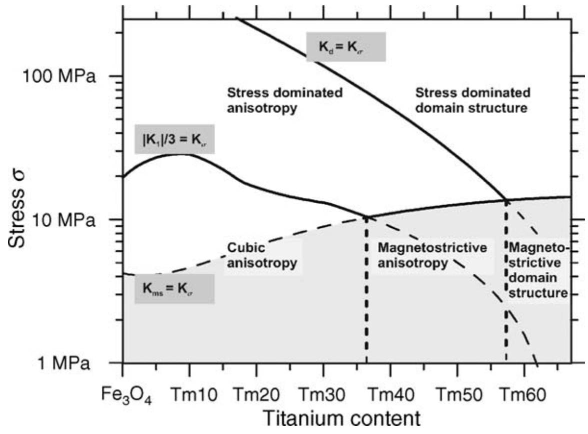

Fig. 1. Comparison of characteristic stress energy K_σ to intrinsic magnetic energies $|K_1|/3$, K_{ms}, and K_d for the titanomagnetite solid solution series. Gray labels indicate the lines along which K_σ is equal to one of these energies. The material constants are collected from Syono and Ishikawa (1963), Klerk et al. (1977), Akimoto et al. (1957), Doraiswami (1947), Syono (1965), Syono et al. (1971), Fletcher and O'Reilly (1974), Carmichael (1982), O'Reilly (1984), Kakol et al. (1991), Hunt et al. (1995). Linear interpolation is used where no specific interpolation formula was suggested in this literature.

while still $Q < 1$, anisotropy is dominated by stress leading to domains oriented along the easy stress axes.

For TMx with $x > 0.37$ one finds $K_{ms} > K_1/3$, and even for low external stress and magnetically soft properties the magnetization structure will reflect magnetostrictive anisotropy which leads to uniaxial domain patterns having minimal magnetostrictive stress. Above $x \approx 57\%$ magnetostrictive energy is even more important than self demagnetization and the material becomes magnetically hard. Accordingly, uniaxial domain patterns prevail at any stress level. Below about 10 MPa the anisotropy may show cubic symmetry due to cubic orientation of the preferred magnetostrictive directions. However, for $\sigma \gg 10$ MPa the domain structure is completely dominated by external stress.

These qualitative predictions agree well with domain observations on single synthetic titanomagnetite grains which clearly exhibit stress controlled uniaxial domain structures (Appel, 1987; Ambatiello and Soffel, 1996). A key to a new interpretation of high M_{rs}/M_s ratios in TM

Table 2
Characteristic scales of volume energy densities in cubic minerals

Energy	Symbol	Characteristic value
Cubic magneto-crystalline anisotropy	K_1	$\|K_1\|/3$
Stress anisotropy	K_σ	$(3/2)\max(\|\lambda_{111}\|, \|\lambda_{100}\|)\,\sigma$
Self-demagnetization	K_d	$(1/2)\mu_0 M_s^2$
Magnetostriction	K_{ms}	$(3/2)(\lambda_{111}^2 c_{44} + \lambda_{100}^2 (c_{11} - c_{12}))$

is the experimental result of Zitzelsberger and Schmidbauer (1996) that the M_{rs}/M_s values for a set of sized ball milled synthetic TM70 samples decrease after annealing. This decrease is even more pronounced for small particles which initially had $M_{rs}/M_s > 0.5$. If high M_{rs}/M_s would have been due to cubic anisotropy, one would expect an increase of M_{rs}/M_s after removing residual stress by annealing. The fact that M_{rs}/M_s decreases therefore indicates that high M_{rs}/M_s ratios may arise from internal stress.

In case of titanomaghemite in MORB older than 10 Myr a recent study yields internal stress estimates in the order of 200 MPa (Hodych and Matzka, 2004). In the following it will be demonstrated that also in young MORB high internal stress effects that the saturation field, necessary to reliably determine M_s, may be well above 1 T. This leads to underestimation of M_s from common measurements of insufficiently saturated samples and results in too high M_{rs}/M_s estimates.

2. Analysis of the approach to saturation

A sufficiently accurate determination of the saturation magnetization in MORB samples may require fields considerably larger than 1 T. To prove this, it is necessary to understand the saturation behavior of magnetic materials.

A magnetic hysteresis loop can be subdivided into two regimes. The irreversible part, where the upper and lower branches are separated and the reversible part at high field values, where both branches coincide. The latter is also denoted as *approach to saturation* part (e.g. Bertotti, 1998). In the reversible part only rotation of the magnetization and slight increase of M_s occurs (para-effect). The magnetization curve in this region can be expressed as

$$M(B) = M_s + \chi B + \alpha B^{\beta}, \tag{1}$$

where χ is the susceptibility of all dia- and paramagnetic components (including the para-effect) and the last term represents an individual approach to saturation law. Both, α and β must be negative to ensure that the approach is from below and vanishes for $B \to \infty$. For homogeneous magnetization rotated by the external field against anisotropy energy within a defect free material one obtains $\beta = -2$ (Akulov, 1931). However, for superparamagnetic ensembles the Langevin function implies a saturation law with $\beta = -1$, and in samples with closely spaced dislocations or other defects, exchange interaction between locally different stress induced deflections leads to even higher values of $-1 < \beta < 0$ (Brown, 1940; Kronmüller and Fähnle, 2003). In rock magnetic evaluations an average value of $\beta = -1$ is often assumed (Nagata, 1961; Dobeneck, 1996).

Here the more general formula (1) is used to determine α and β from experimental data. This provides a simple method to assess whether the maximally applied field is sufficient to reach the *approach to saturation* regime, which definitely is not the case if values of $\beta > 0$ are obtained.

Taking the logarithm of the second derivative of (1) results in

$$\log |M''(B)| = \log |\alpha\beta(\beta - 1)| + (\beta - 2) \log B. \tag{2}$$

By numerically differentiating the high field part of the the measured hysteresis loop it is thus possible to determine α and β from a plot of $\log |M''(B)|$ versus $\log B$. The best fit line has slope $\beta - 2$ and abscissa $\log |\alpha\beta(\beta - 1)|$. If this determination of α and β leads to well defined values below zero, one can determine χ and M_s as slope and abscissa of $M(B) - \alpha B^{\beta}$ versus sufficiently large positive B.

Eq. (2) is conveniently evaluated by defining a smooth interpolation of $M(B)$ and applying the numerical approximation

$$M''(B) \approx \frac{2M(B) - M(B - \Delta B) - M(B + \Delta B)}{\Delta B^2}. \tag{3}$$

Results using different spacings ΔB can be compared in order to discriminate between true measurement noise and discretization artifacts. Both disturbances are inevitable since in the important high field region of the evaluated signal the magnetization change is small on a large background and high field measurements usually require large field increments which can be controlled only with limited precision.

Fig. 2 demonstrates exemplary approach to saturation measurements for ideal magnetite on a diamagnetic sample holder (Fig. 2a), and for a typical MORB sample(Fig. 2b). Even though considerable noise disturbs the linear fit, the magnetite value of $\beta \approx -2$ perfectly agrees with the theoretical value. In contrast, the inferred slope for the MORB sample is considerably smaller and leads to a positive estimate of $\beta \approx 0.5$. This strongly indicates that the true approach to saturation region of the MORB sample is not sufficiently covered by the applied maximum field of 1 T.

3. Application to MORB samples

To substantiate the above single results which coincide with previous observations of Gee and Kent (1995) and Hodych and Matzka (2004), detailed hysteresis mea-

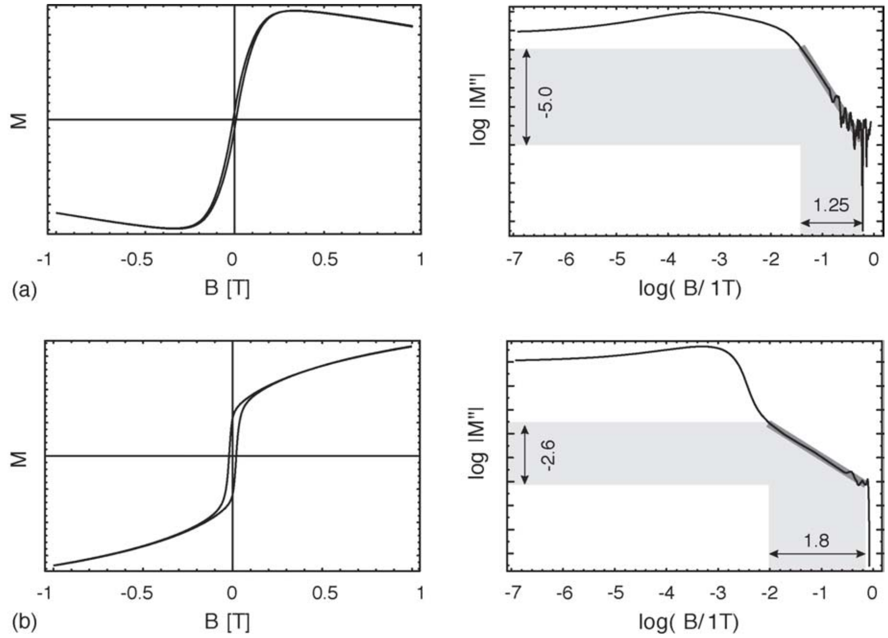

Fig. 2. Approach to saturation analysis of hydrothermal magnetite JM13 (Heider et al., 1987) (a) and MORB sample Xb1 (Kent and Gee, 1996; Fabian, 2003) (b). In both cases the maximum field is 1 T. On the right the plots of a numerical approximation of $\log |M''(B)|$ vs. $\log B$ yield linear high field slopes which correspond to the respective $\beta - 2$.

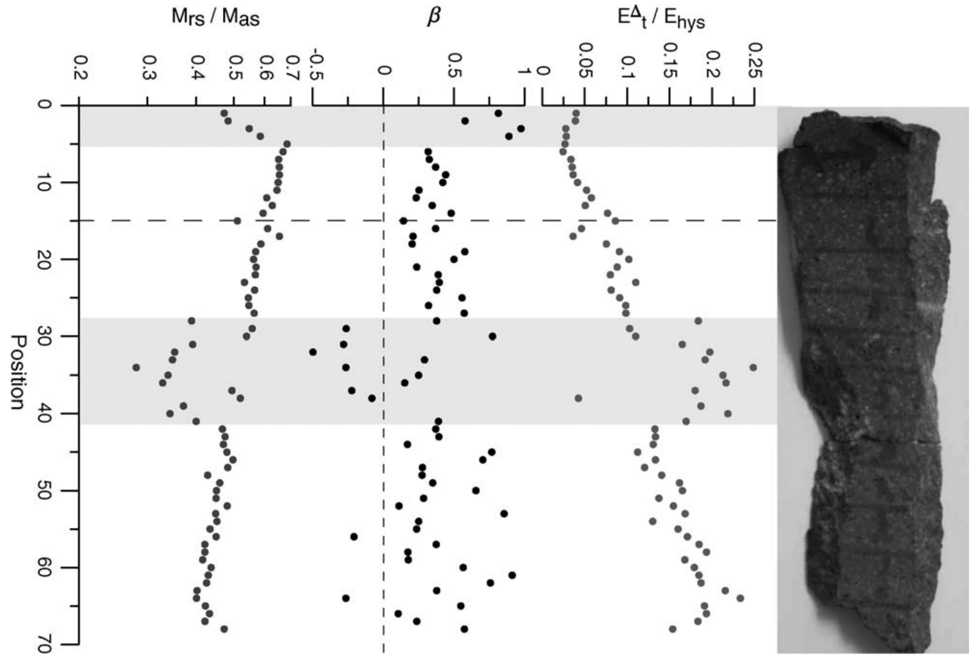

Fig. 3. Transect of hysteresis parameters $M_{\mathrm{rs}}/M_{\mathrm{as}}$, β and $E_t^{\Delta}/E_{\mathrm{hys}}$ along MORB T787-R1 (provided by D. Kent). Sample position varies only approximately linear with distance from the chilled margin. The total length of the MORB specimen is 12 cm. M_{as} denotes the 'apparent' saturation magnetization obtained by subtracting the average slope of the hysteresis loop between 0.7 T and maximum field of 1 T.

surements have been performed on a dredged zero-age MORB from the Juan de Fuca Ridge (pillow T787-R1), kindly provided by D. Kent, and previously investigated by Kent and Gee (1996) and Fabian (2003).

From the slab shown in Fig. 3, 68 samples were taken along a 12 cm transect from the chilled margin (top) to the flow center (bottom). As demonstrated in Gee and Kent (1999), grain size increases steadily from SP-

SD mixtures at the chilled margin to PSD-MD particles along such a transect. This general trend has been confirmed for the sample at hand in Fabian (2003).

All measurements – including those presented in Fig. 2 – were performed at the University of Bremen using a Micromag 2000 AGFM built by Princeton Measurements Inc. to obtain hysteresis loops, saturation initial curves and backfield curves up to fields of maximally 1 T. Hysteresis loops and M_{si}-curves in high fields up to 7 T have been measured using a MPMS-XL built by Quantum Design.

For all 1 T hysteresis loops of the described MORB samples an 'apparent' saturation magnetization M_{as} has been estimated by the instrument standard procedure of subtracting the average slope of the hysteresis loop above 70% of the maximum field. This correction method leads to ratios of $M_{\mathrm{rs}}/M_{\mathrm{as}} > 0.5$ for most positions between 5 and 30 in Fig. 3. On the other hand, the approach to saturation analysis of the previous section mostly results in positive values for β. This strongly indicates that the premise for the application of a reliable slope correction is not fulfilled. Namely the *approach to saturation* regime of the hysteresis loop is not reached within the 1 T field range. That the estimated saturation magnetization M_{as} from a linear interpolation of the last 30% of the upper hysteresis branch in this case considerably underestimates the true M_{s} value can be convincingly demonstrated by comparing it to the M_{s} estimate obtained from a hysteresis loop measured within a field range of 7 T. Fig. 4 shows the result of this comparison for sample Xc1, which approximately corresponds to position 20 in Fig. 3. Here $M_{\mathrm{as}}(1\,\mathrm{T})$ is by nearly 30% lower than the high field M_{s} estimate, which in turn reduces the ratio of $M_{\mathrm{rs}}/M_{\mathrm{as}} \approx 0.5$ to $M_{\mathrm{rs}}/M_{\mathrm{s}} \approx 0.39$. For the 7 T loop a value of $\beta \approx -1.8 \pm 0.2$ is estimated from the high field region above 1 T. Even though the large uncertainty in this value reflects the low signal-to-noise ratio, it definitely proves a sufficiently fast approach to saturation to justify the application of the standard linear M_{s} estimate using fields above 5 T.

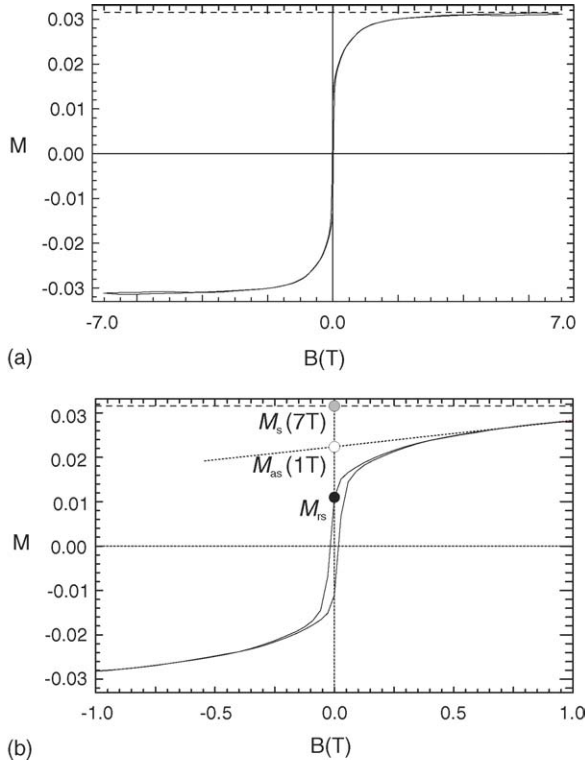

Fig. 4. Hysteresis loop of MORB sample Xc1 measured with a maximum field of 7 T at 300 K. In (a) the slope corrected loop is shown over the full field range. Apparently above 5 T the sample is very well saturated and permits accurate determination of M_{s}. Restricting the plot range to ± 1 T in (b) demonstrates that in this region the sample indeed is far from saturation. Both plots use the same paramagnetic correction and the apparent saturation magnetization M_{as} (1 T) determined from (b) is too low by nearly 30%.

4. Removing the influence of MD remanence

For all hysteresis loops of MORB samples measured up to 7 T in this study, or up to 5 T in previous studies (Gee and Kent, 1995; Matzka et al., 2003), the ratio $M_{\mathrm{rs}}/M_{\mathrm{s}}$ was found to lie below 0.5. Therefore, these results do not provide direct evidence for dominating cubic anisotropy. However, as elaborated by Gee and Kent (1995), electron microscopic images reveal the presence of relatively large MD titanomagnetite grains in all investigated MORB samples. Such grains have the potential to considerably reduce the overall $M_{\mathrm{rs}}/M_{\mathrm{s}}$ and accordingly the observed values still may result from mixing a small MD value $\mu < 0.15$ and a cubic SD value of 0.866 in concentration c. This would result in a total

$$\frac{M_{\mathrm{rs}}}{M_{\mathrm{s}}} = \mu(1 - c) + 0.866c, \tag{4}$$

which for c <50% is smaller than 0.5. The problem therefore is to make sure that the $M_{\mathrm{rs}}/M_{\mathrm{s}}$ ratio of the SD fraction is not larger than 0.5. It is solved here by using the additional, independent grain size parameter $E_t^{\Delta}/E_{\mathrm{hys}}$ to estimate the amount of MD material within each sample (Fabian, 2003).

This parameter measures the energy fraction dissipated by transient *irreversible* processes – such as domain wall nucleation or domain wall pinning – which occur within the self-induced magnetostatic field. Transient irreversible processes are abundant only in MD particles where the self-induced magnetostatic field is sufficient to partially demagnetize the particle. They are completely absent in SD particles.

E_t^Δ is determined by a slightly extended hysteresis measurement scheme (Fabian and von Dobeneck, 1997; Fabian, 2003) which is performed in the following way:

(1) Apply $B = B_{\max}$ and then set $B = 0$ to prepare the sample in the state of saturation remanence M_{rs}.
(2) Measure initial curve plus hysteresis loop starting from this state.

The energy related to the difference between the obtained *saturation initial curve* $M_{si}(B)$ and the upper hysteresis branch $M^+(B)$ is denoted as

$$E_t^\Delta = 2\int_0^{B_{\max}} (M^+(B) - M_{si}(B))\,\mathrm{d}B. \tag{5}$$

It is dissipated solely by transient irreversible processes initiated by the action of the self-demagnetizing field of the sample. The ratio between E_t^Δ and the total hysteresis area

$$E_{hys} = \int_{-B_{\max}}^{B_{\max}} (M^+(B) - M^-(B))\,\mathrm{d}B, \tag{6}$$

determines which fraction of the total energy dissipation results from transient energy dissipation.

The determination of E_t^Δ/E_{hys} is independent of the M_{rs}/M_s determination and the theoretical value of E_t^Δ/E_{hys} for an MD free SD sample is 0 (Fabian and von Dobeneck, 1997; Fabian, 2003). The possible influence of any 'hidden' MD remanence is removed in Fig. 5 by plotting M_{rs}/M_s versus E_t^Δ/E_{hys} for seven samples from different locations along the MORB transect. The linearly extrapolated value of M_{rs}/M_s at $E_t^\Delta/E_{hys} = 0$ now reflects the ideal SD value and permits to decide whether dominant cubic anisotropy is present – even if the titanium content is not perfectly constant across the profile. All samples used in Fig. 5 are taken sufficiently far from the chilled margin to exclude the presence of a significant fraction of superparamagnetic grains which could reduce M_{rs}/M_s without influencing E_t^Δ/E_{hys}. In passing it is noted that in contrast to the interpretation in Zhou et al. (2000), neither viscosity nor shape variation provides any evidence for the presence of a significant fraction of superparamagnetic particles more than 1–2 cm away from the pillow rim (Fabian, 2003).

Clearly the extrapolated SD value of the 7 T M_{rs}/M_s ratios in Fig. 5 perfectly agrees with the value of 0.5 for uniaxial anisotropy and rules out a significant influence of cubic anisotropy. In comparison, the 1 T M_{rs}/M_{as} ratios extrapolate to an SD limit above 0.7 which could not be explained by uniaxial anisotropy. This discrepancy

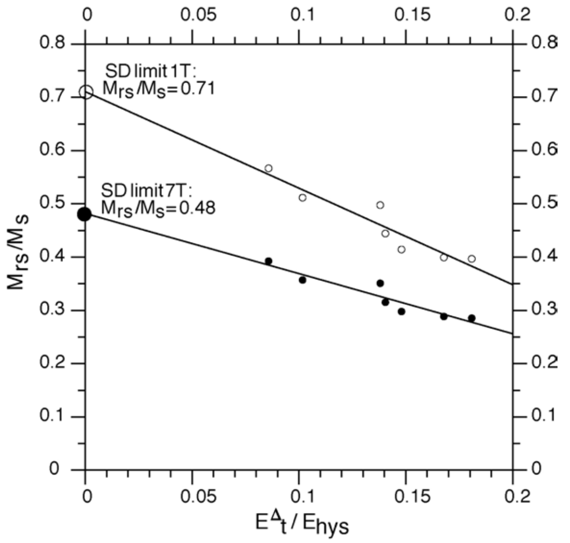

Fig. 5. The plot of M_{rs}/M_s vs. E_t^Δ/E_{hys} shows that for hysteresis loops measured up to 1 T (open circles) the apparent saturation magnetization is considerably lower than for the same samples when measured in fields up to 7 T (full circles). Linear extrapolation to $E_t^\Delta/E_{hys} = 0$ shows that the estimated pure SD value of M_{rs}/M_s does not significantly differ from 0.5, the theoretical value of uniaxial SD ensembles. The extrapolated value for the 1 T loops corresponds well with the extrapolation of Gee and Kent (1999).

vividly illustrates the importance of correctly estimating M_s. By determining β it will be easier to assess the validity of the implicit assumptions made when M_s is determined from the approach to saturation in hysteresis measurements.

5. Discussion

The above presented data establish that M_{rs}/M_s ratios from high field measurements provide no evidence for dominating cubic anisotropy in MORB titanomagnetites. According to the analysis of Gee and Kent (1995), this result is in conflict with the high values of K_1 (or better K') found for Al-substituted titanomagnetite in Özdemir and O'Reilly (1981). However, the determination of K_1 in Özdemir and O'Reilly (1981) also is based on hysteresis loops and for their *approach to saturation* law corresponding to (1) it is a priori assumed that $\beta = -2$. Since this is only valid in the high-field regime of samples where coherent rotation of the magnetic moment is the main mode of magnetization change, and since the applied fields may not have reached into this region, the inferred anisotropy constant may still be influenced by residual stress in the samples.

However, even by assuming that cubic anisotropy is not the main anisotropy term, the reason why MORB samples cannot be saturated in fields below 2 T remains to be understood. Indeed, the most reasonable possibility is that this property originates from stress in combination with the strikingly high magnetostriction of titanomagnetites with high Ti content. The results of Appel (1987), Ambatiello and Soffel (1996), Zitzelsberger and Schmidbauer (1996) coincide well with the energy analysis in Fig. 1 in that stress seems to dominate domain structure and magnetic properties of titanomagnetite already at moderate stress values.

For titanomagnetite samples with sufficiently high, and known Ti content the average magnitude of internal stress $\bar{\sigma}$ can be estimated from the reversible magnetization work U_{rev} (Kersten, 1932; Appel, 1987). By assuming the material parameters $K_1 = -4.0 \times 10^3$ J/m^3, $J_s = \mu_0 M_s = 0.15$ T and $\lambda_{100} = 180 \times 10^{-6}$ (Akimoto et al., 1957; Syono, 1965; Klerk et al., 1977) of TM60 and an average demagnetizing factor of $N = 0.3$, an evaluation of eight 7 T hysteresis loops from different locations across the T787-R1 MORB sample yields $U_{\text{rev}} =$ 3.2–3.7 $\times 10^4$ J/m^3 and accordingly $\bar{\sigma} =$ 220–260 MPa. The error of this estimate, due to uncertainties in the true material constants, is at least $\pm 30\%$. Therefore, and since the true type of internal stress is not known, differences between λ_s and λ_{100} are neglected here. In magnetite, the accuracy of using U_{rev} to determine internal stress is mainly limited by the uncertainty of the average demagnetizing factor N (Hodych, 1990). Here, the energy of the demagnetizing field turns out to be about an order of magnitude smaller than U_{rev} and errors in N lead to only insignificant changes in $\bar{\sigma}$.

The inferred magnitude of internal stress is compatible with estimates for ball milled titanomagnetite (O'Reilly, 1984) and the recent determination of internal stress in titanomaghemite by Hodych and Matzka (2004), but it is considerably higher than the value of $\bar{\sigma} \approx 50$ MPa reported in Appel (1987). The reason for this discrepancy is probably related to the different origin of the basalt samples (Hodych and Matzka, 2004). Internal stress is generated by all processes which influence the local crystal lattice. This includes dislocations, defects and inclusions as well as oxidation fronts or diffusive gradients. The latter may be especially pronounced in the rapidly cooled MOR basalt since during non-equilibrium crystallization, variations in ulvöspinel content and degree of oxidation can be sustained even within single particles (Zhou et al., 1997). The observation of shrinkage cracks due to low temperature oxidation in older ocean floor basalt has been used by Petersen and Vali (1987) to estimate the breaking stress of titanomagnetite to about 500 MPa which may serve as an upper stress limit even for our samples. In young MORB also the effect of rapid cooling may increase the stress due to incompatibility of the basaltic mineral compound created by sequential crystallization. An internal stress of more than 200 MPa also easily explains for the observed B_c values of up to 80 mT for the T787-R1 MORB samples.

A quantitative estimate of the theoretical approach to saturation behavior can be obtained by minimizing the sum of anisotropy, stress and magnetostatic energy

$$K_1(m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2) - (3/2)\lambda_{100}\sigma(mu)^2 - M_s B m v, \tag{7}$$

in function of the unit magnetization vector m. Here v denotes the unit field direction, and the unit vector u is parallel to the axis of uniaxial compression. Two-fold spherical averaging over all possible directions v and u results in a function $F(\sigma, B)$ which gives the degree of saturation for an isotropic ensemble of stressed TM60 particles. The two extreme cases shown in Fig. 6 are easily accessible to numerical calculation. In the first case it is assumed that cubic anisotropy is dominant and the anisotropy constant is set to the maximum literature value of $K_1 = -10.7 \times 10^4$ J/m^3 (Özdemir and O'Reilly, 1981). The second case assumes that stress anisotropy prevails with $\lambda_{100} = 180 \times 10^{-6}$ and $\bar{\sigma} \approx 200$ MPa as model parameters. The results are compared with the upper branch of the hysteresis loop from MORB T787-R1 sample Xb12. Again the stress based interpretation leads to a superior fit which even can be improved by taking into account a stress distribution instead of a single value. The same is not true for the interpretation based on cubic anisotropy. Since the MORB hysteresis loops are usually closed at about 0.3 T, at least the discrepancy in Fig. 6 above this field strength is definitely relevant.

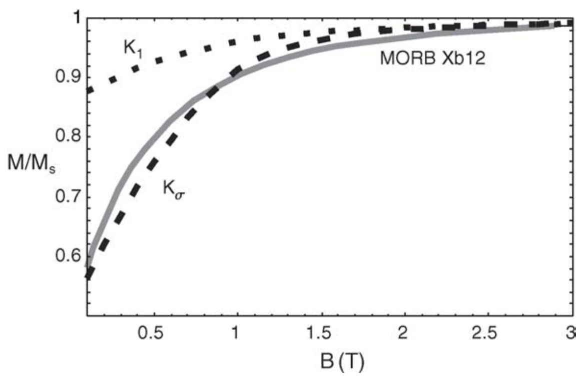

Fig. 6. Comparison between theoretical approach to saturation curves for cubic anisotropy (dotted) using the value $K_1 = -10.7 \times 10^4$J/m^3, stress induced anisotropy (dashed) using $\lambda_{100} = 180 \times 10^{-6}$ and $\bar{\sigma} \approx 200$ MPa, and the measured upper hysteresis branch from MORB T787-R1 sample Xb12 from the vicinity of the rim (solid gray).

6. Conclusions

The previous investigations lead to the following conclusions

- By determining the *approach to saturation* exponent β directly from the hysteresis data, it is possible to detect insufficiently saturated samples and thereby to avoid misinterpretation of M_s. On the other hand, knowledge of β can be used to achieve improved M_s estimates from sufficiently saturated samples.
- MORB samples are typically not sufficiently saturated in commonly used 1 T fields. It has been proved that this has led to considerable overestimation of their M_{rs}/M_s ratio. In high fields of 7 T, MORB samples are sufficiently saturated and all M_{rs}/M_s ratios are below 0.5.
- By plotting M_{rs}/M_s versus E_t^{Δ}/E_{hys} and extrapolating to the ideal SD value of $E_t^{\Delta}/E_{hys} = 0$ it is shown that also the SD fraction in MORB has a M_{rs}/M_s ratio less or equal 0.5.
- It is demonstrated that uniaxial stress anisotropy in titanomagnetite can explain all observed hysteresis properties. The magnitude of internal stress is estimated to about $\bar{\sigma} = 220$–260 MPa. The comparison of measured loops to theoretical approach to saturation curves also favors dominant stress anisotropy rather than cubic anisotropy.

Acknowledgments

I wish to thank D. Kent for many valuable discussions and comments, and for donating the MORB sample from Juan de Fuca Ridge. The hydrothermal magnetite was donated by F. Heider. Helpful reviews by J. Hodych and D. Krása as well as remarks by A. Kosterov and V.P. Shcherbakov are gratefully acknowledged. The MPMS measurements for this research were supported by DFG grant Bl 154/24.

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